STA220H1: The Practice of Statistics I

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January 17, 2023

Please turn on your videos :)

Figure 1: [picture source]

Learning strategy

Wednesday-Friday: watch modules at
https://sta220.utstat.utoronto.ca

Wednesday-Friday: do practice sets, attend TAs office hours if something is not clear

Friday-Monday: do Quiz, attend my office hours on Monday if something is still not clear

Get help: post your questions on Piazza (not my personal email, pls!) or attend office hours

Agenda for today

- Recap: summary statistics, boxplots
- Summarizing one quantitative variable: histogram, standard deviation
- Summarizing relationship between two variables: barplot, scatterplot, correlation

Recap: data

variable

```
sta220.data
```

```
##             student grade
## 1      Jenny Holder    77
## 2        Tammy Snow    88
## 3     Victoria Hall    90
## 4    Saoirse Spence    86
## 5       Raja Cooper    94
## 6  Nicolas Roberson    68
## 7    Finnley Wright    85
## 8       Nate Mcgrath    93
## 9    Joshua Pollard    82
```

observations

Recap: mean

- Measures the **central tendency** of a data set

```
sta220.data$grade
```

$x_1 \quad x_2 \quad x_3 \qquad\qquad\qquad\quad x_n$

$n = \#\text{obs}$

```
## [1] 77 88 90 86 94 68 85 93 82
```

x_i

```
mean(sta220.data$grade)
```

```
## [1] 84.77778
```

$$\text{Mean} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Recap: median

- Also measures the **central tendency** of a data set
- If we were to sort all of the values, then the **median** is the value in the middle

```
sort(sta220.data$grade)
```

$X_{(1)}$ $X_{(2)}$ $X_{(3)}$ $X_{(n)} = X_{(9)}$

```
## [1] 68 77 82 85 86 88 90 93 94
```

$X_{(1)} = \min$

$X_{(n)} = \max$

```
median(sta220.data$grade)
```

```
## [1] 86
```

Recap: median

- Sometimes we need to use **interpolation** (when n even)

```
sort(sta220.data$grade[1:8])
```

```
## [1] 68 77 85 86 88 90 93 94
```

$$\frac{86+88}{2}$$

```
median(sta220.data$grade[1:8])
```

```
## [1] 87
```

Recap: first and third quartiles

- To find the **first quartile** we travel quarter (1/4) way through the sorted list
- To find the **third quartile** we travel three quarters (3/4) way through the sorted list

```
sort(sta220.data$grade)
```

Q1 median Q3

```
## [1] 68 77 82 85 86 88 90 93 94
```

```
quantile(sta220.data$grade)
```

```
##   0%  25%  50%  75% 100%
##   68   82   86   90   94
```

Recap: first and third quartiles

- Sometimes we need to use **interpolation** (when $n - 1$ is not divisible by 4)

```
sort(sta220.data$grade[1:8])
```

$p = 2.75$ median $Q_1 = 77 + 0.75\,(85-77)$

```
## [1] 68 77 85 86 88 90 93 94
```

1 n $\boxed{p = 1 + (n-1)\cdot \cancel{0.25}}$ 0.75

```
quantile(sta220.data$grade[1:8])
```

$(n-1)$

```
##    0%   25%   50%   75%  100%
## 68.00 83.00 87.00 90.75 94.00
```

Recap: boxplot

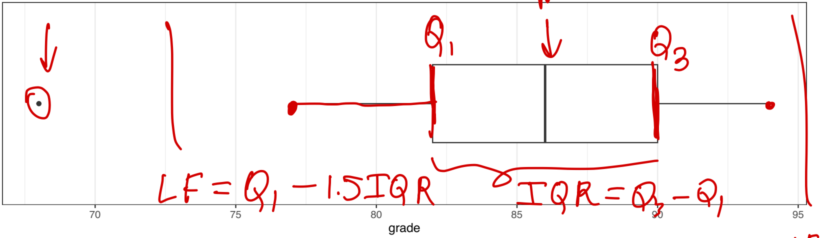
```
quantile(sta220.data$grade)
```

```
##    0%   25%   50%   75%  100%
##    68    82    86    90    94
```

$IQR = 90 - 82 = 8$

$Q_1 - 1.5 \cdot IQR = 82 - 1.5 \cdot 8$

median



Q_1 Q_3

$LF = Q_1 - 1.5 IQR$

$IQR = Q_3 - Q_1$

$UF = Q_3 + 1.5 IQR$

$LF \leq W_1$ $W_2 \leq UF$

Recap: boxplot

- No observations in $[LF, Q_1]$ range $\Rightarrow$ no lower whisker
- No observations in $[Q_3, UF]$ range $\Rightarrow$ no upper whisker

outlier

```
grade = c(40,80,80,80,80,85,86,94,95,100,100,100,100)
quantile(grade)
```

80

quantile

Q_1, Q_3

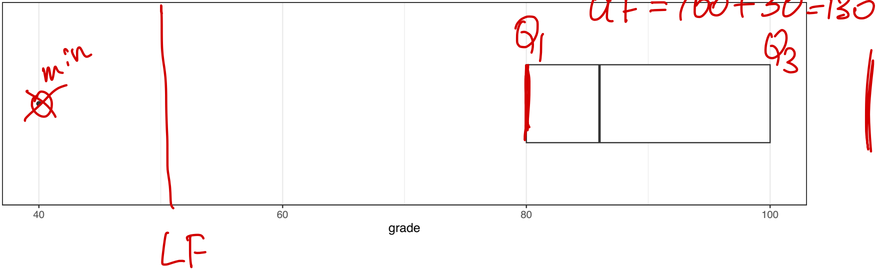
$\boxed{IQR(\ldots)}$

```
##   0%  25%  50%  75% 100%
##   40   80   86  100  100
```

$IQR = 20$

$LF = 80 - 30 = 50$

$UF = 100 + 30 = 130$



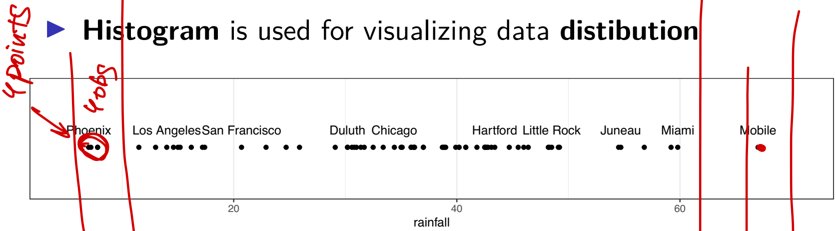
Plots: histogram

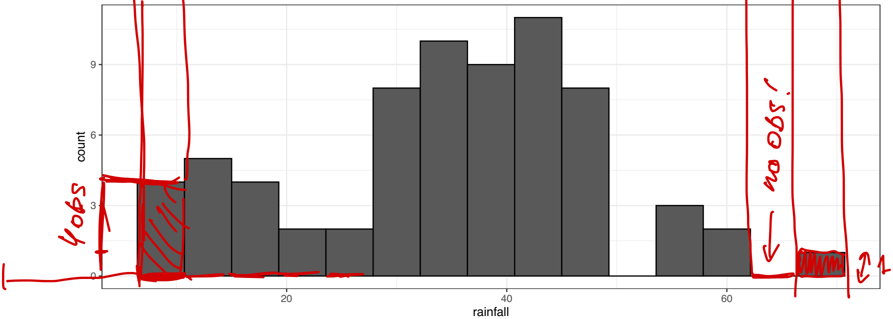
Data set: the rainfall level in inches for 69 United States cities

	rainfall
Mobile	67.0
Juneau	54.7
Phoenix	7.0
Little Rock	48.5
Los Angeles	14.0
Sacramento	17.2
San Francisco	20.7
Denver	13.0
Hartford	43.4
Wilmington	40.2
Washington	38.9
Jacksonville	54.5
Miami	59.8
Atlanta	48.3
Honolulu	22.9

Plots: histogram

- **Histogram** is used for visualizing data **distibution**

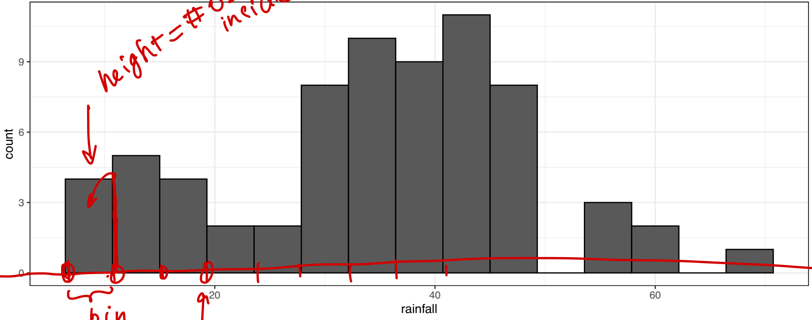




Plots: histogram

- X-axis is split in **bins**, they should be mutually exclusive and exhaustive
- **Breaks (cutpoints)** are the values that define the beginnings and the ends of the bins
- **Counts (frequencies)** are numbers of data points in each bin (height of each bar)

height = # observation inside bin N1

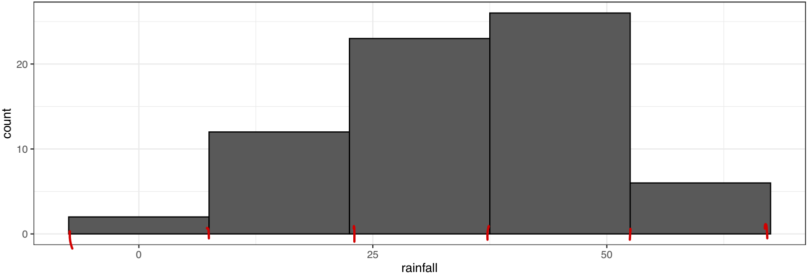


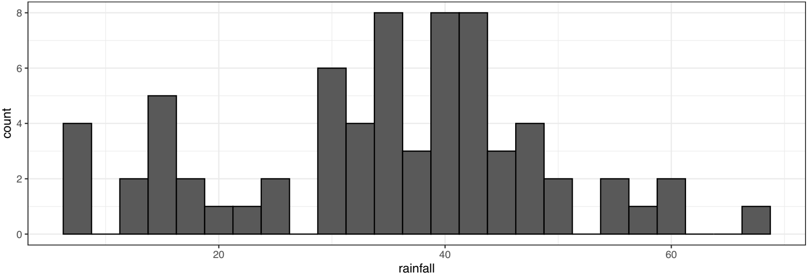
bin

cutpoint

Plots: histogram

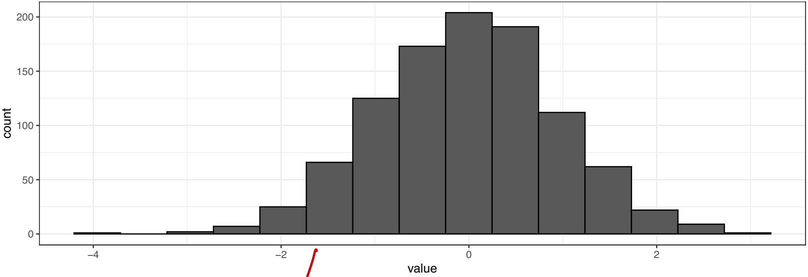
- The appearance of histogram **depends on the cutpoints**

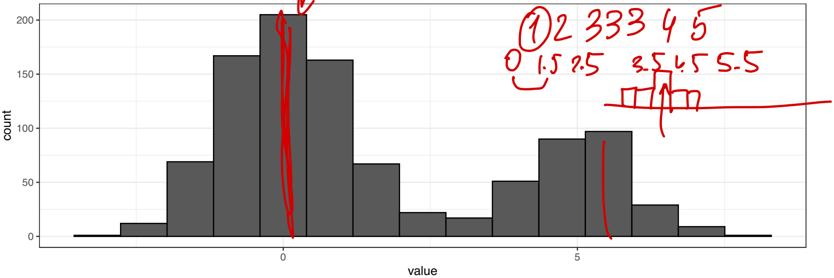




Plots: histogram

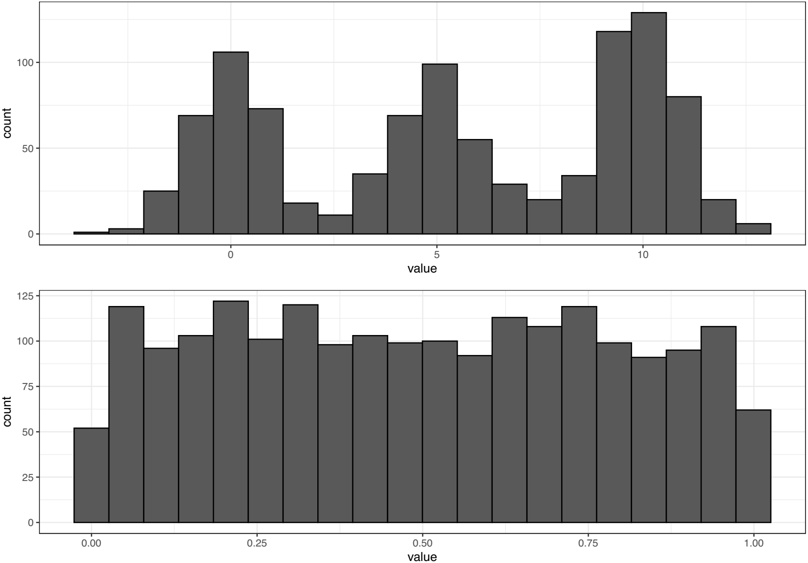
- **Mode** - the peak of the distribution
- Histogram can be **unimodal**, **bimodal**, **multimodal**, **uniform**





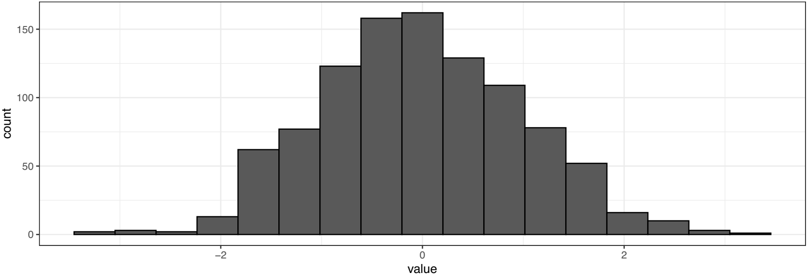
Plots: histogram

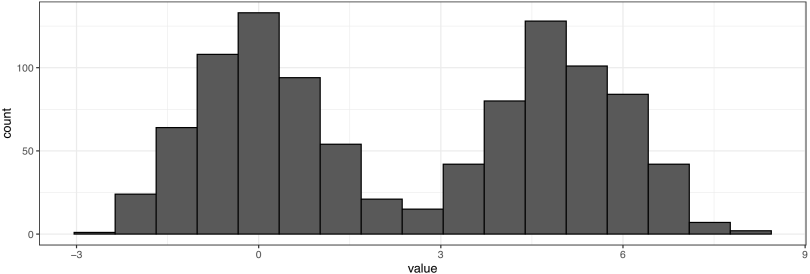
- **Mode** - the peak of the distribution
- Histogram can be **unimodal**, **bimodal**, **multimodal**, **uniform**



Plots: histogram

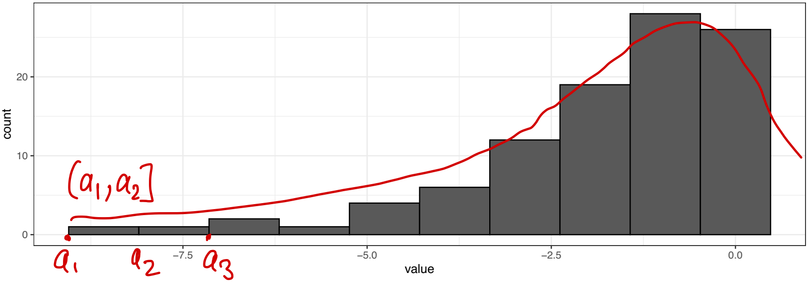
- Histogram can be **symmetric**, **left-skewed** (long left tail), **right-skewed** (long right tail)

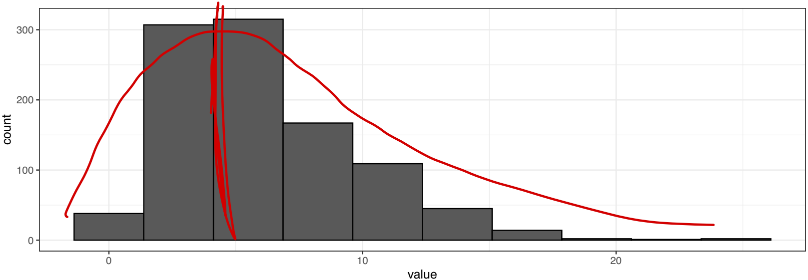




Plots: histogram

- Histogram can be **symmetric**, **left-skewed** (long left tail), **right-skewed** (long right tail)

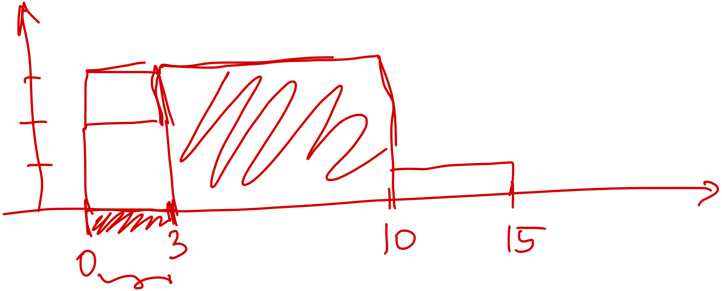




Exercise

3 1 2 3

For a sample 11,1,2,6,6,6 plot the histogram with cutpoints -1, 0,3,10,15. How many bars are there? How tall is each bar?



Summary statistics: standard deviation

There are several ways to measure the **spread of the data**

$$IQR = Q_3 - Q_1$$

max min

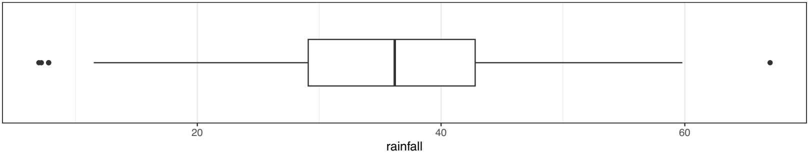
$$range = x_{(n)} - x_{(1)}$$

```
IQR(precip.data$rainfall)
```

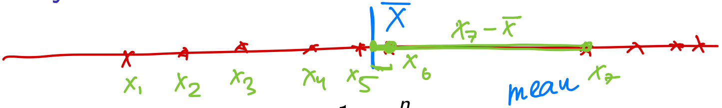
```
## [1] 13.7
```

```
max(precip.data$rainfall) - min(precip.data$rainfall)
```

```
## [1] 60
```



Summary statistics: standard deviation



$$variance = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = s_x^2$$

$\Sigma(x_i - \bar{x}) = 0$

$$standard\ \ deviation = \sqrt{variance} = s_x$$

```
var(precip.data$rainfall)
```

```
## [1] 190.5252
```

```
sd(precip.data$rainfall)
```

```
## [1] 13.80309
```

Exercise

$$variance = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = s_x^2$$

Compute standard deviation of the following values:

3, 10, 5, 6, 10, 8?

```
vec = c(3, 10, 5, 6, 10, 8)
summary(vec)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    3.00    5.25    7.00    7.00    9.50   10.00
```

$n = 6$

1) $x_i - \bar{x}$ $\quad (-4, 3, -2, -1, 3, 1)$

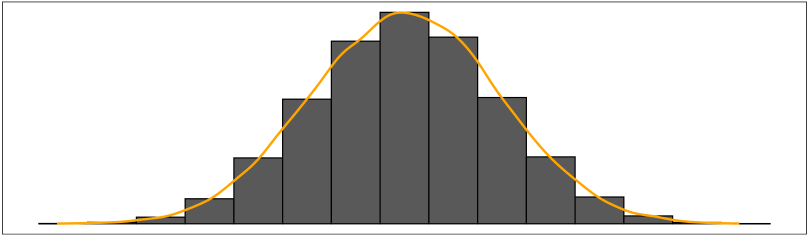
2) $(x_i - \bar{x})^2$ $\quad 16, 9, 4, 1, 9, 1$

3) $(16+9+4+1+9+1)/5 = \ldots = var$

4) $\sqrt{}$

Summary statistics: standard deviation

There is an **empirical rule** for **symmetric, unimodal, bell-shaped** distributions.



Summary statistics: standard deviation

- **68%** of the data lies in $[\bar{x} - s_x, \bar{x} + s_x]$
- **95%** of the data lies in $[\bar{x} - 2 \cdot s_x, \bar{x} + 2 \cdot s_x]$
- **99.7%** of the data lies in $[\bar{x} - 3 \cdot s_x, \bar{x} + 3 \cdot s_x]$

$50\% + \frac{68\%}{2} = 84\%$

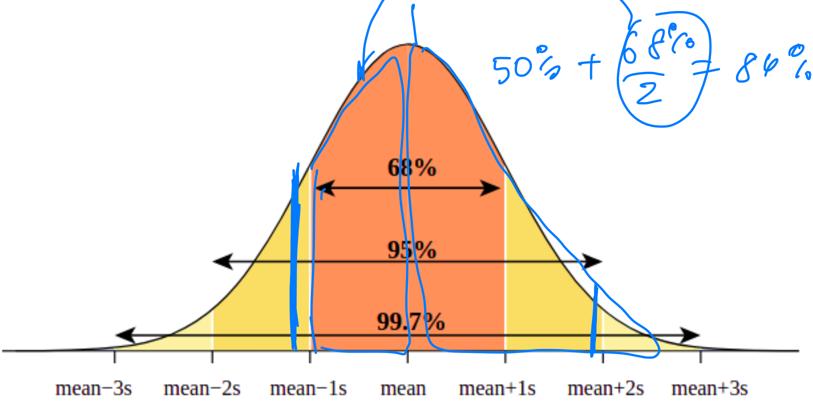


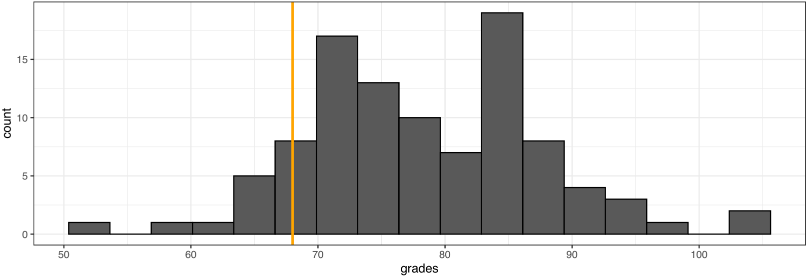
Figure 2: [picture source]

How bad is my midterm score of 68?

Option 1: use a histogram to compare your score to other students.

```
summary(grades)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   51.00   72.00   77.50   78.27   84.25  103.00
```



How bad is my midterm score of 68?

Option 2: quantify your relative performance using z-score.

- **z-score** is an adjustment of a data value to get its position in a data set
- It tells you how many standard deviations a data value is away from its mean

$$z = \frac{\overset{68}{x} - \bar{x}}{s_x}$$

```
(mygrade - mean(grades))/sd(grades)
```

```
## [1] -1.126134
```

Data summary: one quantitative variable

- Compute **numerical summary (summary statistics)** - mean, minimum, maximum, range, median, quartiles, IQR, standard deviation
- Summarize using **plots** - histogram and boxplot

Data summary: one categorical variable

- **Numerical summary** is very limited - frequencies, relative frequencies
- Summarize using **plots** - barplot, piechart

Data summary: one categorical variable

Data set: an experiment was conducted to measure effectiveness of various feed supplements on the growth rate of 71 chickens

weight	feed
179	soybean
160	soybean
136	soybean
227	soybean
217	soybean
168	soybean
108	soybean
124	soybean
143	soybean
140	soybean
309	linseed
229	linseed
181	linseed
141	linseed
260	linseed

Numerical summary: distribution

- **Distribution** describes how data are divided between different possible values
- **Frequencies** measure how many observations are in each category

```r
tab = table(chick.data$feed)
tab
```

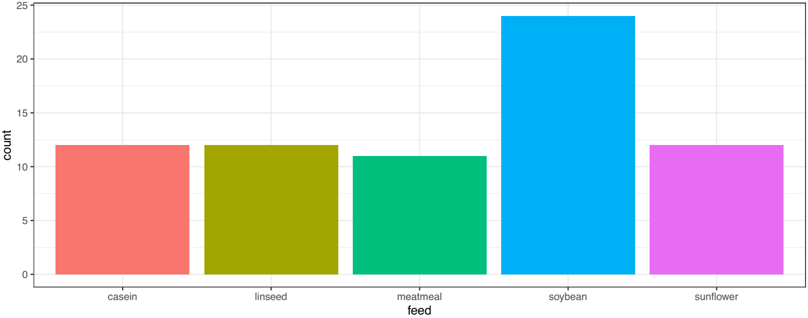
```
##
##    casein   linseed  meatmeal   soybean sunflower
##        12        12        11        24        12
```

/71 /71 /71 /71 /71

f_1/n

Plots: barplot

- In a sense, this is an analogue of a histogram



Numerical summary: distribution

- **Distribution** describes how data are divided between different possible values
- **Relative frequencies** measure proportion of observations in each category

```
prop.table(tab)
```

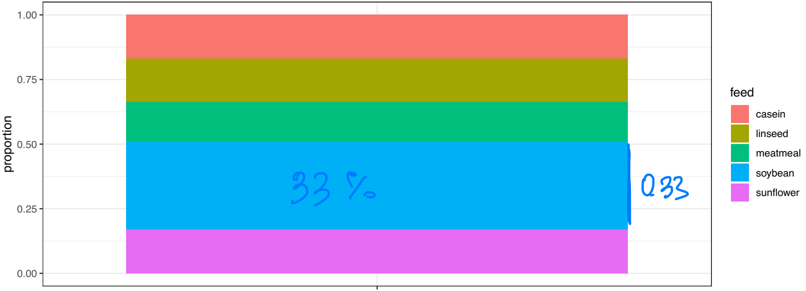
```
##
##    casein   linseed  meatmeal   soybean sunflower
## 0.1690141 0.1690141 0.1549296 0.3380282 0.1690141
```

+ + + + = 1

$f_1/n \quad f_2/n \quad \ldots \quad f_5/n$

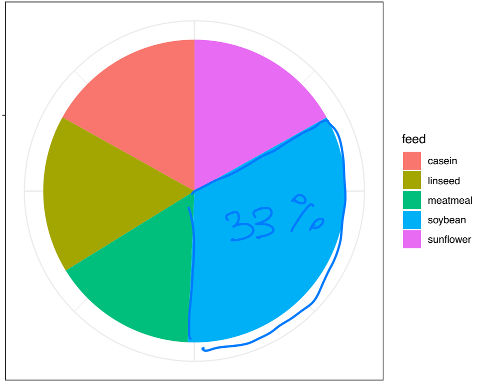
Plots: stacked barplot

- All proportions add up to one!



Plots: piechart

- Size of each slice illustrates the proportion of a category



Exercise

You get the distribution (frequencies) of pets in the building you live. The information was collected among n students. Can you estimate n?

$0.1n$

```
##
##     cat     dog    fish hamster  iguana    none
##      15      12       1       4       3      15
```

$f_1 \quad f_2 \quad f_3 \quad \ldots \quad f_6$

$$\boxed{f_1 + f_2 \ldots + f_6 = n}$$

$f_1/n \quad f_2/n \quad \ldots \quad f_6/n$

0.1	0.1	0.2	0.2	0.3	0.1

Data summary: quantitative vs quantitative variables

- **Summary statistics** - correlation
- Use **plots** - scatterlplot

Data summary: quantitative vs quantitative variables

Data set: 1078 measurements of a father's height and his son's height.

X_{1078}, Y_{1078}

$(y_1 - \bar{y})^2$ $(x_1 - \bar{x})$

$(y_2 - \bar{y})^2$ $(x_2 - \bar{x})^2$

y_3 x_3

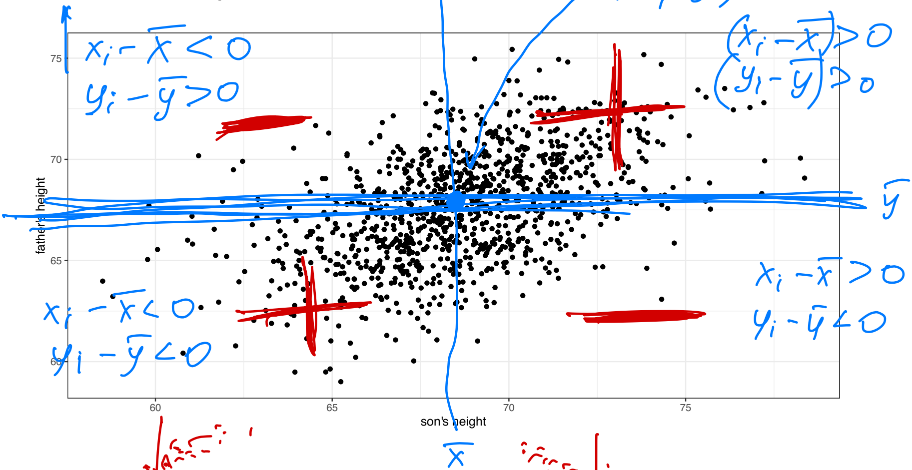
fheight	sheight
65.04851	59.77827
63.25094	63.21404
64.95532	63.34242
65.75250	62.79238
61.13723	64.28113
63.02254	64.24221
65.37053	64.08231
64.72398	63.99574
66.06509	64.61338
66.96738	63.97944
59.00800	65.24451
62.93203	65.35102
63.67063	65.67992
64.07386	65.43664
64.68851	65.29391

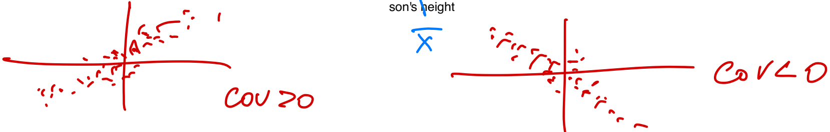
Plots: scatterplot

$$\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$$

- Is there any relation between variables?

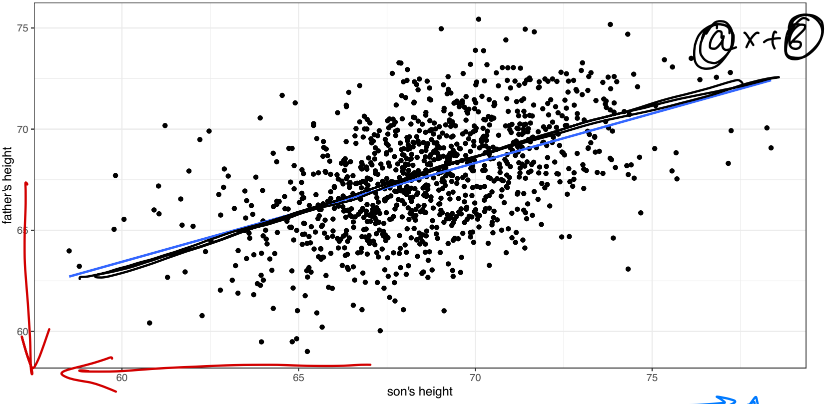
$(\bar{x}, \bar{y})$



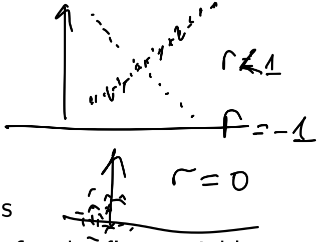


Plots: scatterplot

- There seems to be a positive relationship: taller father ⇒ taller son



Summary statistics: covariance



Can we quantify the trend?

- n will denote the number of observations
- $x_1, x_2, ..., x_n$ will denote the observations for the first variable
- $y_1, y_2, ..., y_n$ will denote the observations for the second variable

$$covariance = \frac{1}{n-1}\sum_{i=1}^{n}(\underbrace{\overset{100 x_i}{x_i} - \overset{100 \cdot \bar{x}}{\bar{x}}}_{100})(\underbrace{y_i - \bar{y}}_{100}) = cov_{xy}$$

x_i meters $\longrightarrow$ $x_i \cdot 100$ (cm)

$100 \cdot cov_{xy}$ $\quad 100 y_i$

Summary statistics: covariance

- Positive **covariance** ⇒ the variables tend to both increase together
- Negative **covariance** ⇒ one variable tends to increase when the other decreases
- But it depends on the scale of variables!

```
cov(father.son.data$sheight, father.son.data$fheight)
```

```
## [1] 3.873333
```

Summary statistics: correlation

- **Correlation** refers to the scaled form of covariance
- Correlation value is between -1 and 1

$$correlation = \frac{cov_{xy}}{s_x \cdot s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} = r_{xy}$$

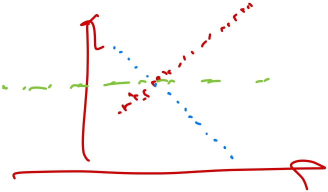
Summary statistics: correlation

Can we quantify the trend?

- If there is a perfect linear relationship, e.g. $y_i = a \cdot x_i + b$, then correlation is 1 (if $a > 0$) or -1 (if $a < 0$)

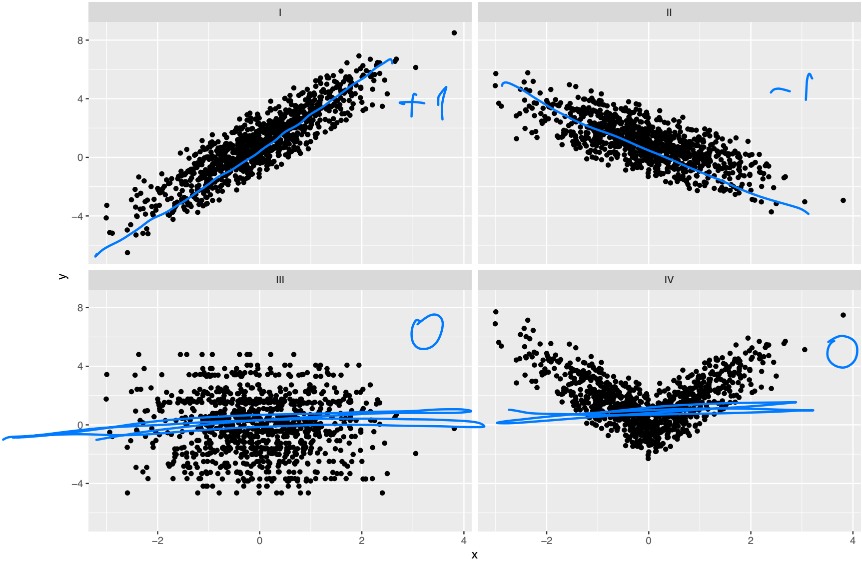
```
cor(father.son.data$sheight, father.son.data$fheight)
```

```
## [1] 0.5013383
```



Exercise

What is the correlation (close to 1,-1 or 0)?



Data summary: categorical vs quantitative variables

- Compute **summary statistics** - within each category
- Use **plots** - boxplot

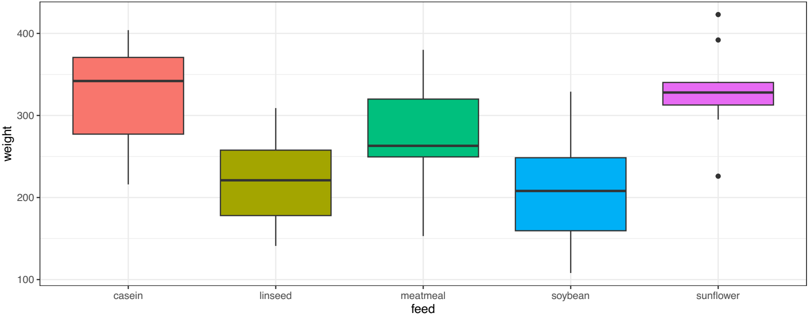
Summary statistics

- You can compute summary statistics, e.g. mean, median and sd, within each category

feed	min	max	mean	median	Q1	Q3	sd
casein	216	216	323.5833	342	277.25	~~277.25~~	64.43384
linseed	141	141	218.7500	221	178.00	~~178.00~~	52.23570
meatmeal	153	153	276.9091	263	249.50	~~249.50~~	64.90062
soybean	108	108	210.5000	208	159.50	~~159.50~~	64.23124
sunflower	226	226	328.9167	328	312.75	~~312.75~~	48.83638

Plots: boxplot

- Use x-axis for different categories
- This method is good, but sometimes it is really hard to say if the difference is significant



Data summary: categorical vs categorical variables

- **Numerical summary** is very limited - frequencies and relative frequencies
- Use **plots** - barplot

Data summary: categorical vs categorical variables

Data set: provides information on the fate of 891 passengers on the fatal maiden voyage of the ocean liner "Titanic", summarized according to economic status (class), sex, age and survival.

PassengerId	Sex	Age	Class	Survived
1	male	22	3	No
2	female	38	1	Yes
3	female	26	3	Yes
4	female	35	1	Yes
5	male	35	3	No
6	male	NA	3	No
7	male	54	1	No
8	male	2	3	No
9	female	27	3	Yes
10	female	14	2	Yes
11	female	4	3	Yes
12	female	58	1	Yes
13	male	20	3	No
14	male	39	3	No
15	female	14	3	No

Numerical summary: joint distribution

Is it true that rich people (e.g. 1st class passengers) survived more often that poor people (e.g. 3rd class passengers)?

```
table(titanic.data$Class)
```

```
##
##   1   2   3
## 216 184 491
```

```
table(titanic.data$Survived)
```

```
##
##  No Yes
## 549 342
```

Numerical summary: joint distribution

- **Joint distribution** is the frequency/relative frequency of observations for a combination of two variables

```
tab = table(titanic.data$Class, titanic.data$Survived)
tab
```

Survived

f_{11}/n

f_{12}/n

```
##
##      No Yes
##   1  80 136
##   2  97  87
##   3 372 119
```

Class

f_{32}

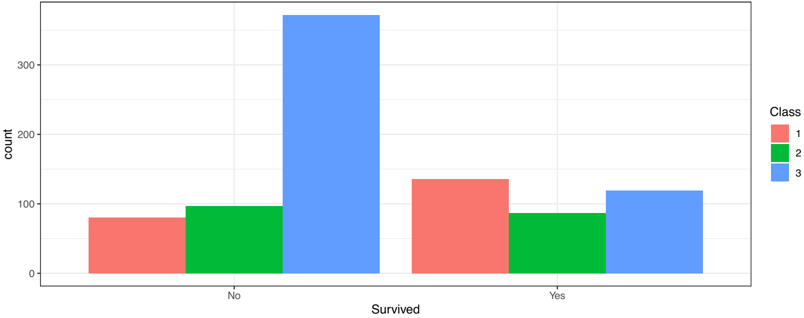
```
ptab = prop.table(tab)
ptab
```

80/891

```
##
##             No        Yes
##   1 0.08978676 0.15263749
##   2 0.10886644 0.09764310
##   3 0.41750842 0.13355780
```

Plots: barplot

- There are many 3rd class passengers that did not survive
- But it is hard to compare as there were many people who did not survive



Numerical summary: marginal distribution

- **Marginal distribution** is the frequency/relative frequency of only one variable

```
addmargins(tab)
```

```
##
##          No Yes Sum
##   1      80 136 216
##   2      97  87 184
##   3     372 119 491
##   Sum   549 342 891
```

Numerical summary: conditional distribution

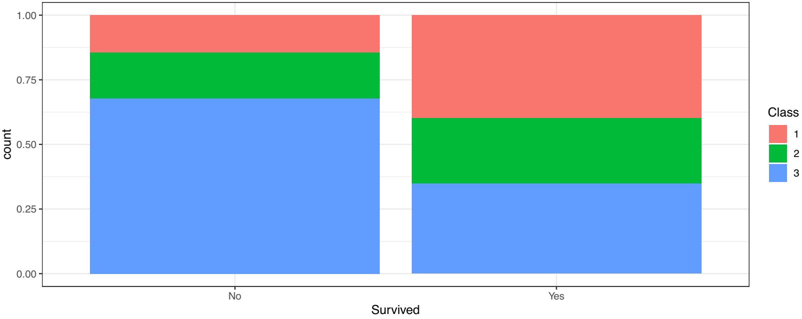
- **Conditional distribution** is the distribution of one variable within a fixed value of a second value
- Comparing conditional distributions for each cetegory can tell if there is any relationship between two variables

```
##
##        No Yes
##   1    80 136
##   2    97  87
##   3   372 119
##   Sum 549 342
```

```
##
##              No       Yes
##   1   0.1457195 0.3976608
##   2   0.1766849 0.2543860
##   3   0.6775956 0.3479532
##   Sum 1.0000000 1.0000000
```

Plots: stacked barplot

- Two variables are **independent** if conditional distribution of one variable is the same for all values of the other variable



Exersice

Find conditional distribution of Sex and Survived variables. Do you think there is any relationship?

```
##
##           No Yes
##   female  81 233
##   male   468 109
```

TO DO

1. Module 1. Summarizing Data: One variable and Module 1. Summarizing Data: Relationships Between Variables
2. Quiz 2 due Monday (January 23) @ 11:59 PM (EST)
3. Practice Problem Set 2